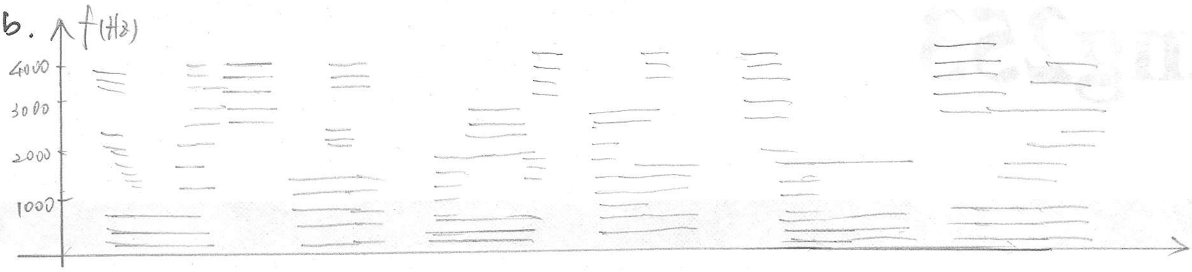
ECE 438 HW7 Solution.

1. a. wideband.

b.



c. Each division mark is $\approx \frac{2\,\text{sec}}{20} = 100\,\text{ms}$

Count $\approx$ 12 pitch periods in one dimension.

$$T_p \approx \frac{100\,\text{ms/division}}{12\,\text{periods/division}} = 8.3\,\text{ms/period}$$

d. $F_1 \approx 300\,\text{Hz}$

$F_2 \approx 2200\,\text{Hz}$

$F_3 \approx 2700\,\text{Hz}$

e. Such a high F_2 and F_3 would indicate IY (beet) /i/

2. a. $\dfrac{50 \text{ samples/period}}{10{,}000 \text{ samples/sec}} = 5\text{ms/period}$

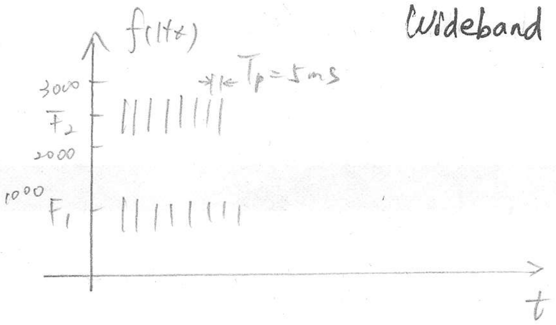
b. In the DTFT, $\omega = \pi$ corresponds to $f = \frac{f_s}{2} = 5000$.

$F_1 = \left(\frac{30^\circ}{180^\circ}\right)(5000 \text{ Hz}) = 833 \text{ Hz}$

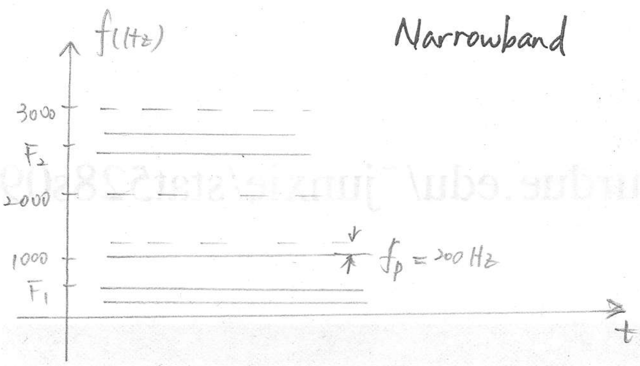
$F_2 = \left(\frac{90^\circ}{180^\circ}\right)(5000 \text{ Hz}) = 2500 \text{ Hz}$

and $|F_1| > |F_2|$, Since F_1 poles are closer to unit circle.

c. Wideband



d. Narrowband



3.a. linearity. Let $v[n] = aX[n] + by[n]$

$$V(w,n) = \sum_k v[k]\, w[n-k]\, e^{-jwk}$$

$$= \sum_k [aX[k] + by[k]]\, w[n-k]\, e^{-jwk}$$

$$= a\sum_k x[k]\, w[n-k]\, e^{-jwk} + b\sum_k y[k]\, w[n-k]\, e^{-jwk}$$

$$= aX(w,n) + bY(w,n)$$

b. shifting. Let $v[n] = x[n-n_0]$

$$V(w,n) = \sum_k x[k-n_0]\, w[n-k]\, e^{-jwk}$$

$$= \sum_{k'} x[k']\, w[n-k'-n_0]\, e^{-jw(k'+n_0)}$$

$$= e^{-jwn_0} \sum_k x[k]\, w[n-n_0-k]\, e^{-jwk}$$

$$= e^{-jwn_0} X(w, n-n_0)$$

c. shifting. Let $v[n] = x[n]e^{jw_0 n}$

$$V(w,n) = \sum_k x[n] e^{jw_0 k}\, w[n-k]\, e^{-jwk}$$

$$= \sum_k x[k]\, w[n-k]\, e^{-j(w-w_0)k}$$

$$= X(w-w_0, n)$$

d. $x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(\mu) e^{j\mu n}\, d\mu$

$w[n-k] = \frac{1}{2\pi}\int_{-\pi}^{\pi} W(\theta) e^{j\theta(n-k)}\, d\theta$

$$X(w,n) = \sum_k x[k]\, w[n-k]\, e^{-jwk}$$

$$= \sum_k \frac{1}{2\pi}\int_{-\pi}^{\pi} X(\mu)\, e^{j\mu k}\, d\mu \int_{-\pi}^{\pi} \frac{1}{2\pi} \cdot (\theta) e^{j\theta(n-k)}\, d\theta\, e^{-jwk}$$

$$= \left(\frac{1}{2\pi}\right)^2 \sum_k \int_{-\pi}^{\pi}\int_{-\pi}^{\pi} X(\mu)\, W(\theta)\, e^{j\theta n} e^{j(\mu-\theta-w)k}\, d\mu\, d\theta$$

$$= \left(\frac{1}{2\pi}\right)^2 \int_{-\pi}^{\pi}\int_{-\pi}^{\pi} X(\mu)W(\theta)e^{j\theta n} \underbrace{\sum_k e^{j(\mu-\omega)k} e^{-j\theta k}} d\mu\, d\theta$$

$$= DTFT_\theta\{e^{j(\mu-\omega)k}\} = 2\pi\delta(\theta-\mu+\omega)$$
$$= 2\pi\delta(\mu-\theta-\omega)$$

$$= \frac{1}{2\pi}\int_{-\pi}^{\pi} W(\theta)e^{j\theta n} \int_{-\pi}^{\pi} X(\mu)\,\delta(\mu-(\theta+\omega))\,d\mu\, d\theta$$

$$= \frac{1}{2\pi}\int_{-\pi}^{\pi} W(\theta)\, e^{j\theta n}\, X(\theta+\omega)\,d\theta$$

4. $x[n] = \begin{cases} \cos\frac{\pi n}{8} & n<0 \\ \cos\frac{\pi n}{3} & n\geq 0 \end{cases}$ $\quad w[n] = \begin{cases} 1 & |n|<25 \\ 0 & \text{else} \end{cases}$

$$X(\omega,n) = \sum_{k=-\infty}^{\infty} x[k]\,w[n-k]\,e^{-j\omega k}$$

i. $n<-25$ $\quad X(\omega,n) = \sum_{k=n-24}^{n+24} x[k]e^{-j\omega k}$

$$= \sum_{k=n-24}^{n+24} \cos\frac{\pi k}{8}\, e^{-j\omega k}$$

Let $k' = k-(n-24)$

$$= \sum_{k'=0}^{48} \cos\left(\frac{\pi(k'+(n-24))}{8}\right) e^{-j\omega(k'+(n-24))}$$

$$= e^{-j\omega(n-24)} \sum_{k=0}^{48} \left\{\frac{1}{2}e^{j\pi(k+(n-24))/8} + \frac{1}{2}e^{-j\pi(k+(n-24))/8}\right\} e^{-j\omega k}$$

$$= e^{-j(\omega-\frac{\pi}{8})(n-24)} \sum_{k=0}^{48} \frac{1}{2} e^{-j(\omega-\frac{\pi}{8})k} + e^{-j(\omega+\frac{\pi}{8})(n-24)} \sum_{k=0}^{48} \frac{1}{2} e^{-j(\omega+\frac{\pi}{8})k}$$

$$= e^{-j(\omega-\frac{\pi}{8})(n-24)} \left(\frac{1}{2}\right) \frac{\sin((\omega-\frac{\pi}{8})/\frac{49}{2})}{\sin((\omega-\frac{\pi}{8})/\frac{1}{2})} e^{-j(\omega-\frac{\pi}{8})24}.$$

$$+ e^{-j(\omega+\frac{\pi}{8})(n-24)} \left(\frac{1}{2}\right) \frac{\sin((\omega+\frac{\pi}{8})/\frac{49}{2})}{\sin((\omega+\frac{\pi}{8})/\frac{1}{2})} e^{-j(\omega+\frac{\pi}{8})24}.$$

$$= \frac{1}{2} e^{-j(w-\frac{\pi}{8})n} \frac{\sin((w-\frac{\pi}{8})\frac{49}{2})}{\sin((w-\frac{\pi}{8})\frac{1}{2})} + \frac{1}{2} e^{-j(w+\frac{\pi}{8})n} \frac{\sin((w+\frac{\pi}{8})\frac{49}{2})}{\sin((w+\frac{\pi}{8})\frac{1}{2})}$$

ii $n > 25$

$$X(w,n) = \frac{1}{2} e^{-j(w+\frac{\pi}{3})n} \frac{\sin((w-\frac{\pi}{3})\frac{49}{2})}{\sin((w-\frac{\pi}{3})\frac{1}{2})}$$

$$+ \frac{1}{2} e^{-j(w-\frac{\pi}{3})n} \frac{\sin((w+\frac{\pi}{3})\frac{49}{2})}{\sin((w-\frac{\pi}{3})\frac{1}{2})}$$

iii $n=0$.

$$X(w,0) = \sum_{k=-24}^{24} x[k] e^{-jwk}$$

$$= \sum_{k=-24}^{-1} \cos(\frac{\pi k}{8}) e^{-jwk} + \sum_{k=0}^{24} \cos(\frac{\pi k}{3}) e^{-jwk}$$

$$= \sum_{k=0}^{24} \cos(\frac{\pi k}{8}) e^{jwk} - 1 + \sum_{k=0}^{24} \cos(\frac{\pi k}{3}) e^{-jwk}$$

$$= \sum_{k=0}^{24} \frac{1}{2} \left\{ e^{j(w+\frac{\pi}{8})k} + e^{j(w-\frac{\pi}{8})k} + e^{-j(w-\frac{\pi}{3})k} + e^{-j(w+\frac{\pi}{3})k} \right\} - 1$$

$$X(w,0) = \frac{1}{2} e^{j(w+\frac{\pi}{8})12} \left\{ \frac{\sin((w+\frac{\pi}{8})\frac{25}{2})}{\sin((w+\frac{\pi}{8})\frac{1}{2})} \right\} + \frac{1}{2} e^{j(w-\frac{\pi}{8})12} \left\{ \frac{\sin((w-\frac{\pi}{8})\frac{25}{2})}{\sin((w-\frac{\pi}{8})\frac{1}{2})} \right\}$$

$$+ \frac{1}{2} e^{-j(w+\frac{\pi}{3})12} \left\{ \frac{\sin((w+\frac{\pi}{3})\frac{25}{2})}{\sin((w+\frac{\pi}{3})\frac{1}{2})} \right\} + \frac{1}{2} e^{-j(w-\frac{\pi}{3})12} \left\{ \frac{\sin((w-\frac{\pi}{3})\frac{25}{2})}{\sin((w-\frac{\pi}{3})\frac{1}{2})} \right\} - 1$$

b.