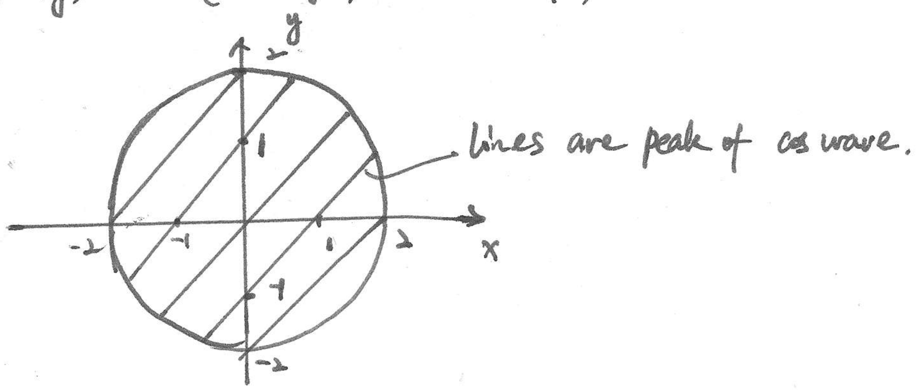
ECE 438 HW8. Soln

1. a. $f(x,y) = \begin{cases} \cos(2\pi(x-y)) & , x^2+y^2<4 \\ 0 & , \text{else} \end{cases}$

i. $f(x,y) = \cos(2\pi(x-y))\,\text{circ}(\frac{x}{4}, \frac{y}{4})$



ii $f(x,y) = \cos(2\pi(x-y))\,\text{circ}(\frac{x}{4}, \frac{y}{4})$

iii $\cos(2\pi(x-y)) = \frac{1}{2}(e^{j2\pi(x-y)} + e^{-j2\pi(x-y)})$

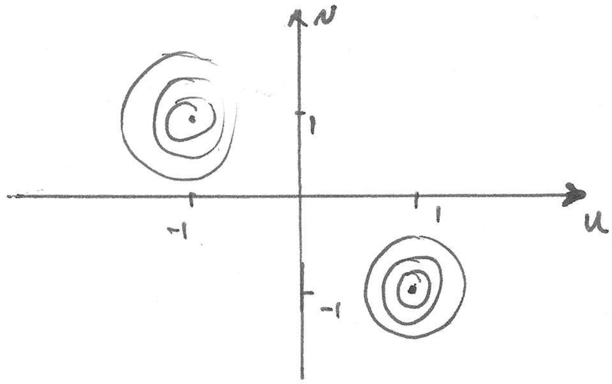
$\longleftrightarrow \frac{1}{2}(\delta(u+1, v-1) + \delta(u-1, v+1))$

$\text{circ}(\frac{x}{4}, \frac{y}{4}) \longleftrightarrow 16\,\text{jinc}(4u, 4v)$

$\therefore F(u,v) = \frac{1}{2}(\delta(u+1, v-1) + \delta(u-1, v+1)) ** 16\,\text{jinc}(4u, 4v)$

$= 8\,\text{jinc}(4(u+1), 4(v-1)) + 8\,\text{jinc}(4(u-1), 4(v+1))$

iv



b. i. sketch has been provided.

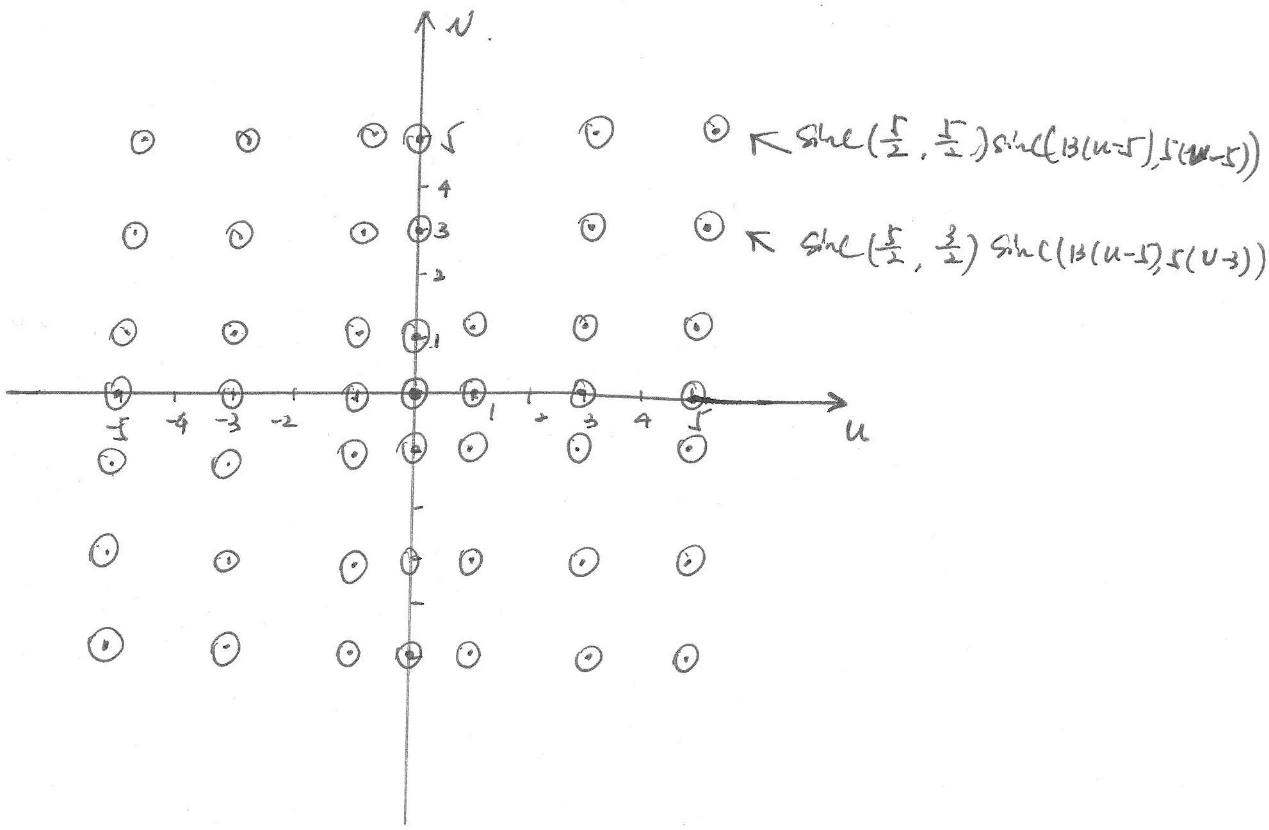
ii $f(x,y) = rep_{1,1}\{rect(2x,2y)\} \cdot rect(\frac{x}{13}, \frac{y}{5})$

iii $F(u,v) = comb_{1,1}\{\frac{1}{4} sinc(\frac{u}{2}, \frac{v}{2})\} ** \{65\, sinc(13u, 5v)\}$

$$F(u,v) = \frac{65}{4} \sum_m \sum_n sinc(\frac{m}{2}, \frac{n}{2})\, \delta(u-m, v-n) ** sinc(13u, 5v)$$

$$= \frac{65}{4} \sum_m \sum_n sinc(\frac{m}{2}, \frac{n}{2})\, sinc(13(u-m), 5(v-n))$$

iv.



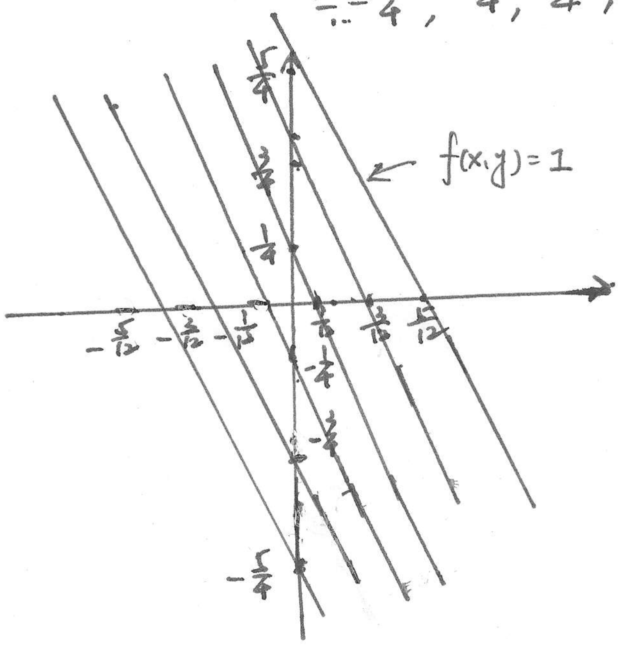
2. $f(x,y) = 1 + \cos(2\pi(3x+y))$

a. $f(x,y) = 1$ $\qquad \cos(2\pi(3x+y)) = 0$

$2\pi(3x+y) = \frac{\pi}{2} + k\pi \qquad k = 0, \pm 1, \pm 2, \cdots$

$3x+y = \frac{2k+1}{4}$

$= \cdots -\frac{3}{4}, -\frac{1}{4}, \frac{1}{4}, \frac{3}{4}, \frac{5}{4} \cdots$

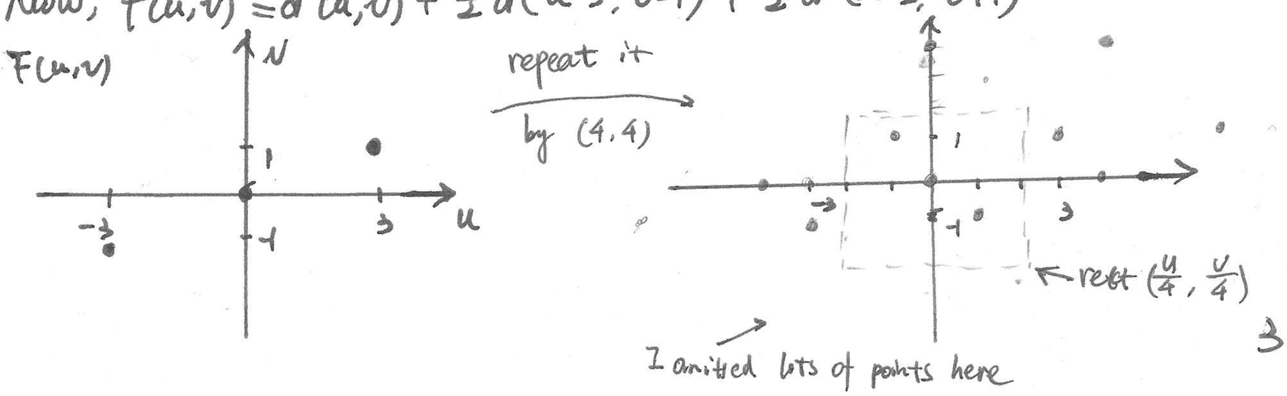


b. $f_r(x,y) = f_s(x,y) ** \mathrm{sinc}(4x, 4y)$ where $f_s(x,y) = \mathrm{comb}_{\frac{1}{4},\frac{1}{4}}(f(x,y))$

$F_r(u,v) = 16\, \mathrm{rep}_{4,4}\{F(u,v)\} \frac{1}{16} \mathrm{rect}(\frac{u}{4}, \frac{v}{4})$

$= \mathrm{rep}_{4,4}\{F(u,v)\} \mathrm{rect}(\frac{u}{4}, \frac{v}{4})$

Now, $F(u,v) = \delta(u,v) + \frac{1}{2}\delta(u-3, v-1) + \frac{1}{2}\delta(u+3, v+1)$

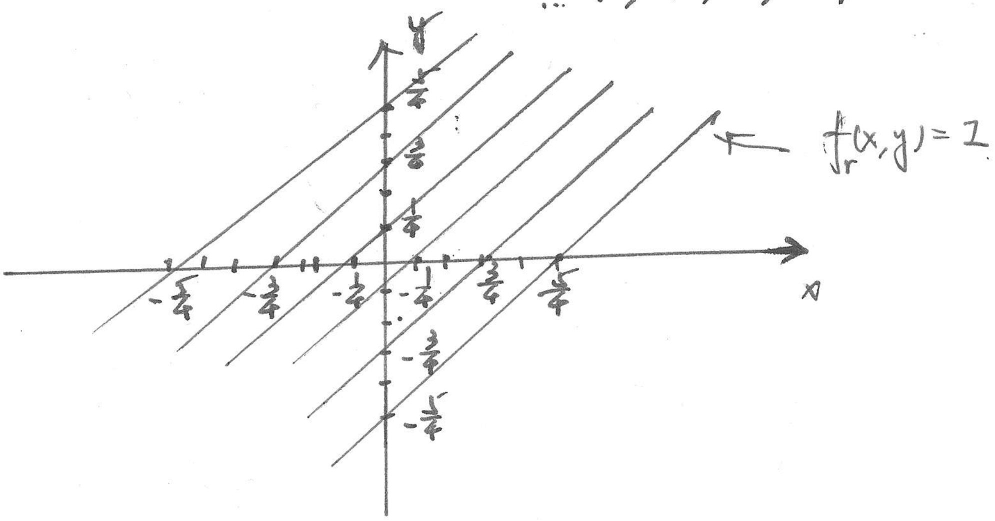


Therefore, $F_r(u,v) = \delta(u,v) + \frac{1}{2}\delta(u-1,v) + \frac{1}{2}\delta(u+1,v-1)$

$\Rightarrow f_r(x,y) = 1 + \cos(2\pi(x-y))$

c. $f_r(x,y) = 1$

$$1+\cos(2\pi(x-y)) = 1$$
$$\cos(2\pi(x-y)) = 0$$
$$2\pi(x-y) = \frac{\pi}{2} + k\pi \qquad k = 0, \pm 1, \pm 2, \dots$$
$$x - y = \frac{2k+1}{4}$$
$$= \dots -\frac{3}{4}, -\frac{1}{4}, \frac{1}{4}, \frac{3}{4}, \dots$$



3. a. $h(m,n) = -\frac{1}{8}\delta(m+1,n-1) + \frac{1}{2}\delta(m,n-1) - \frac{1}{8}\delta(m-1,n-1)$
$-\frac{1}{4}\delta(m+1,n) + \delta(m,n) - \frac{1}{4}\delta(m,n-1)$
$-\frac{1}{8}\delta(m+1,n+1) + \frac{1}{2}\delta(m,n+1) - \frac{1}{8}\delta(m-1,n+1)$

$y(m,n) = -\frac{1}{8}x(m+1,n-1) + \frac{1}{2}x(m,n-1) - \frac{1}{8}x(m-1,n-1)$
$-\frac{1}{4}x(m+1,n) + x(m,n) - \frac{1}{4}x(m,n-1)$
$-\frac{1}{8}x(m+1,n+1) + \frac{1}{2}x(m,n+1) - \frac{1}{8}x(m-1,n+1)$

b.

$$
\begin{matrix}
0 & 0 & 0 & 0 & -\frac{1}{8} & \frac{1}{2} & -\frac{1}{8} & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -\frac{1}{8} & \frac{1}{8} & \frac{10}{8} & \frac{1}{8} & -\frac{1}{8} & 0 & 0 & 0 \\
0 & 0 & -\frac{1}{8} & \frac{1}{8} & \frac{7}{8} & \frac{10}{8} & \frac{7}{8} & \frac{1}{8} & -\frac{1}{8} & 0 & 0 \\
0 & -\frac{1}{8} & \frac{1}{8} & \frac{7}{8} & \frac{8}{8} & 1 & \frac{8}{8} & \frac{7}{8} & \frac{1}{8} & -\frac{1}{8} & 0 \\
-\frac{1}{8} & \frac{1}{8} & \frac{7}{8} & \frac{8}{8} & 1 & 1 & 1 & \frac{8}{8} & \frac{7}{8} & \frac{1}{8} & -\frac{1}{8} \\
-\frac{3}{8} & 1 & \frac{8}{8} & 1 & 1 & 1 & 1 & 1 & \frac{8}{8} & 1 & -\frac{3}{8} \\
-\frac{1}{2} & \frac{3}{2} & 1 & 1 & 1 & 1 & 1 & 1 & 1 & \frac{3}{2} & -\frac{1}{2} \\
-\frac{1}{2} & \frac{3}{2} & 1 & 1 & 1 & 1 & 1 & 1 & 1 & \frac{3}{2} & -\frac{1}{2} \\
-\frac{1}{2} & \frac{3}{2} & 1 & 1 & 1 & 1 & 1 & 1 & 1 & \frac{3}{2} & -\frac{1}{2} \\
-\frac{3}{8} & \frac{8}{8} & \frac{6}{8} & \frac{6}{8} & \frac{6}{8} & \frac{6}{8} & \frac{6}{8} & \frac{6}{8} & \frac{6}{8} & \frac{8}{8} & -\frac{3}{8} \\
-\frac{1}{8} & \frac{3}{8} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{3}{8} & -\frac{1}{8}
\end{matrix}
$$

c. $h_1(m) = \{-\frac{1}{4}, 1, -\frac{1}{4}\} \Leftrightarrow H_1(\mu) = -\frac{1}{4}e^{-j\mu(-1)} + e^{-j\mu(0)} - \frac{1}{4}e^{-j\mu(1)}$

$h_2(n) = \{\frac{1}{2}, 1, \frac{1}{2}\} \Leftrightarrow H_2(\nu) = \frac{1}{2}e^{-j\nu(-1)} + e^{-j\nu(0)} + \frac{1}{2}e^{-j\nu(1)}$

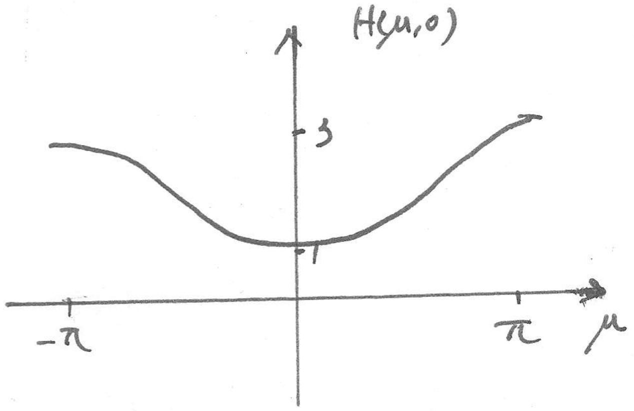
$H(\mu,\nu) = H_1(\mu,\nu)\, H_2(\mu,\nu) = (1 - \frac{1}{2}\cos\mu)(1 + \cos\nu)$

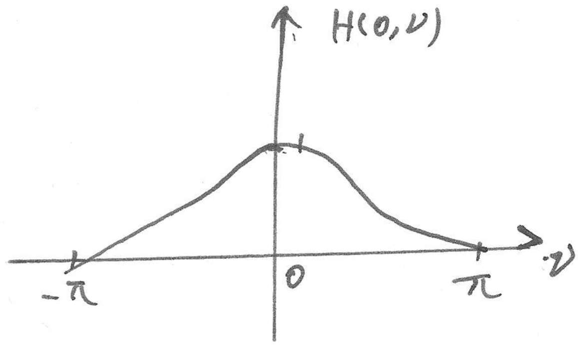
d. $H(\mu, 0) = 2[1 - \frac{1}{2}\cos\mu]$

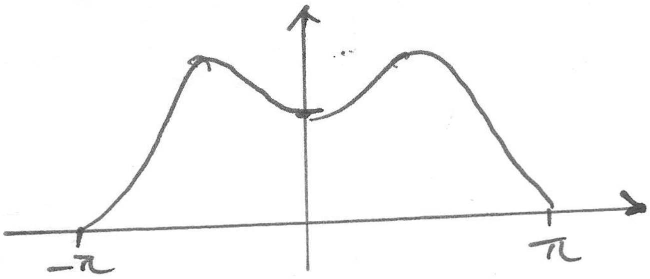
$H(0, \nu) = \frac{1}{2}[1 + \cos\nu]$

$H(\mu,\mu) = [1 - \frac{1}{2}\cos\mu][1 + \cos\mu] = \frac{3}{4} + \frac{1}{2}\cos\mu - \frac{1}{4}\cos 2\mu$

$H(\mu, -\mu) = H(\mu, \mu)$







e. $H(u, 0)$ shows the responce to sinusoids in ↔ direction

$H(0, \ )$ — ↕ direction

$H(u, u)$ — ↗ direction

$H(u, -u)$ — ↖ direction

$H(u, 0)$ indicates an amplification of high horizontal frequencies, which shows up as an overshoot on either side of vertical edges.

$H(0, v)$ indicates an attenuation of high frequences in vertical direction. which is why there's a blurriy of the bottom horizontal edge.

$H(u, u)$ and $H(+u, -u)$ show a slight amplification of mid-range diagonal frequencies and attenuation of high dig. frequencies, which is why there are blurred diagonal edges with a slight overshoot.