ECE 438 HW6 Soln

1. a.

n	...	-3	-2	-1	0	1	2	3	4	5	...
$x[n]$	...	0	-2	0	1	0	-2	0	0	0	...
$y[n]$	...	0	-2	-4	-1	2	-1	-4	-2	0	...

b. $h[n] = \delta[n] + 2\delta[n-1] + \delta[n-2]$

$$y[n] = h[n] * x[n] = \{\delta[n] + 2\delta[n-1] + \delta[n-2]\} * \{-2\delta[n+2] + \delta[n] - 2\delta[n-2]\}$$

$$= -2\delta[n+2] + \delta[n] - 2\delta[n-2] - 4\delta[n-1] + 2\delta[n-1] - 4\delta[n-3]$$
$$- 2\delta[n] + \delta[n-2] - 2\delta[n-4]$$

$$= -2\delta[n+2] - 4\delta[n+1] - \delta[n] + 2\delta[n-1] - \delta[n-2]$$
$$- 4\delta[n-3] - 2\delta[n-4]$$

c. i. if $x[n] = e^{j\omega n}$ then $y[n] = H(\omega)e^{j\omega n}$

$$y[n] = H(\omega)e^{j\omega n} = e^{j\omega n} + 2e^{j\omega(n-1)} + 2e^{j\omega(n-2)}$$

$$H(\omega) = 1 + 2e^{-j\omega} + e^{-j2\omega}$$

ii. $h[n] = \delta[n] + 2\delta[n-1] + \delta[n-2]$

$\delta[n] \leftrightarrow 1$

$\delta[n-1] \leftrightarrow e^{-j\omega}$

$\delta[n-2] \leftrightarrow e^{-j2\omega}$

$$H(\omega) = 1 + 2e^{-j\omega} + e^{-j2\omega}$$

We get the same answer with i.

$$x[n] = -2\delta[n+2] + \delta[n] - 2\delta[n-2]$$

$$X(\omega) = -2e^{j2\omega} + 1 - 2e^{-j2\omega}$$

$$Y(w) = X(w)\,H(w)$$
$$= (-2e^{j2w} + 1 - 2e^{-j2w})(1 + 2e^{-jw} + e^{-j2w})$$
$$= -2e^{j2w} - 4e^{jw} - 1 + 2e^{-jw} - e^{-j2w} - 4e^{-j3w} - 2e^{-j4w}$$

$Y(w) = \sum_n y[n]\,e^{-jwn}$, simply identify terms $y[n]$ in the seriers

n	-2	-1	0	1	2	3	4
$y[n]$	-2	-4	-1	2	-1	-4	-2

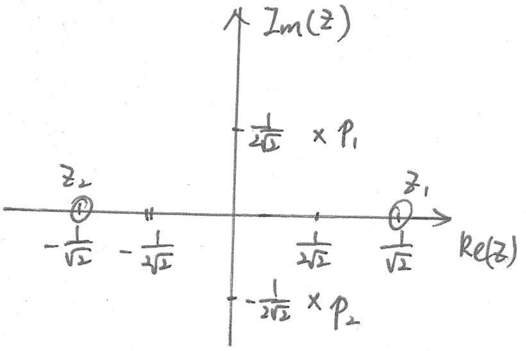
d $y[n]$ from (a), (b), (c) gets the same results.

2.
$$H(z) = \frac{1 - \frac{1}{2}z^{-2}}{1 - \frac{1}{\sqrt{2}}z^{-1} + \frac{1}{4}z^{-2}} = \frac{(z + \frac{1}{\sqrt{2}})(z - \frac{1}{\sqrt{2}})}{\left(z - (\frac{1}{2\sqrt{2}} + j\frac{1}{2\sqrt{2}})\right)\left(z - (\frac{1}{2\sqrt{2}} - j\frac{1}{2\sqrt{2}})\right)}$$

Zeros: $z_1 = \frac{1}{\sqrt{2}}$, $z_2 = -\frac{1}{\sqrt{2}}$

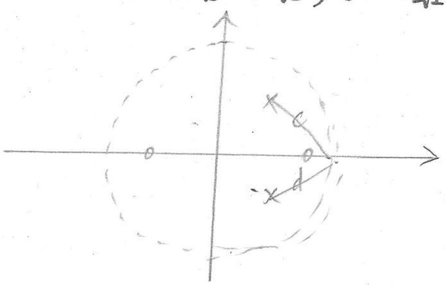
poles: $p_1 = \frac{1}{2\sqrt{2}} + j\frac{1}{2\sqrt{2}}$ $\quad p_2 = \frac{1}{2\sqrt{2}} - j\frac{1}{2\sqrt{2}}$

a.



b. $w = 0$ $\quad \left|H(e^{j0})\right| = \left|H(z=1)\right| = \left|\frac{(1+\frac{1}{\sqrt{2}})(1-\frac{1}{\sqrt{2}})}{\left(1-(\frac{1}{2\sqrt{2}}+j\frac{1}{2\sqrt{2}})\right)\left(1-(\frac{1}{2\sqrt{2}}-j\frac{1}{2\sqrt{2}})\right)}\right| \approx 0.821$

$$\angle H(e^{j0}) = \angle a + \angle b - \angle c - \angle d$$
$$= 0$$



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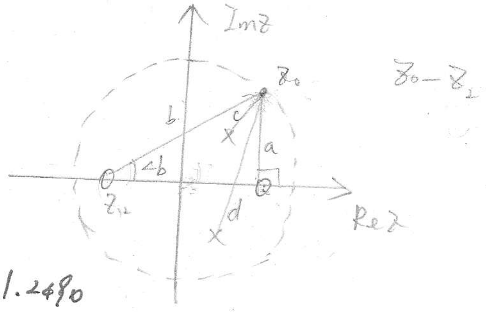
$w=\frac{\pi}{4}$, $z=e^{j\frac{\pi}{4}} = \frac{1}{\sqrt{2}}+j\frac{1}{\sqrt{2}}$

$$\left|H(e^{j\frac{\pi}{4}})\right| = \left|H(z=\tfrac{1}{\sqrt{2}}+j\tfrac{1}{\sqrt{2}})\right| = \left|\frac{(\frac{1}{\sqrt{2}}+j\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}})(\frac{1}{\sqrt{2}}+j\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{2}})}{(\frac{1}{\sqrt{2}}+j\frac{1}{\sqrt{2}}-\frac{1}{2\sqrt{2}}-j\frac{1}{2\sqrt{2}})(\frac{1}{\sqrt{2}}+j\frac{1}{\sqrt{2}}-\frac{1}{2\sqrt{2}}+j\frac{1}{2\sqrt{2}})}\right|$$

$$= 2.$$



$\angle a = \frac{\pi}{2}$, $\angle b = \arctan\left(\frac{\frac{1}{\sqrt{2}}}{\frac{2}{\sqrt{2}}}\right) = 0.4636$

$\angle c = \frac{\pi}{4}$, $\angle d = \arctan\left(\frac{\frac{1}{\sqrt{2}}+\frac{1}{2\sqrt{2}}}{1-\sqrt{2}-\frac{1}{2\sqrt{2}}}\right) = 1.2490$

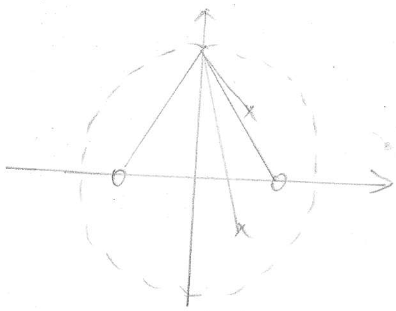
$2 \times \frac{1}{\sqrt{2}}$

$\angle H(e^{j\frac{\pi}{4}}) = \angle a + \angle b - \angle c - \angle d = 0$

$W=\frac{\pi}{2}$, $z=e^{j\frac{\pi}{2}} = j$

$$\left|H(e^{j\frac{\pi}{2}})\right| = \left|H(z=j)\right| = \left|\frac{(j+\frac{1}{\sqrt{2}})(j-\frac{1}{\sqrt{2}})}{(j-\frac{1}{2\sqrt{2}}-j\frac{1}{2\sqrt{2}})(j-\frac{1}{2\sqrt{2}}+j\frac{1}{2\sqrt{2}})}\right|$$

$$= 1.455$$



$\angle a = \arctan(-\frac{1}{\sqrt{2}}) + \pi = 2.186$

$\angle b = \pi - \angle a = 0.9553$

$\angle c = \arctan\left(\frac{1-\frac{1}{2\sqrt{2}}}{-\frac{1}{2\sqrt{2}}}\right) + \pi = 2.0713$

$\angle d = \arctan\left(\frac{1+\frac{1}{2\sqrt{2}}}{1-\frac{1}{2\sqrt{2}}}\right) + \pi = 1.8263$

$\angle H(e^{j\frac{\pi}{2}}) = \angle a + \angle b - \angle c - \angle d = -0.7563$

$\omega = \frac{3}{4}\pi \qquad z = e^{j\frac{3\pi}{4}} = -\frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}}$

$$|H(e^{j\frac{3\pi}{4}})| = |H(z=-\tfrac{1}{\sqrt{2}}+j\tfrac{1}{\sqrt{2}})| = \frac{\left|(-\frac{1}{\sqrt{2}}+j\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}})(-\frac{1}{\sqrt{2}}+j\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{2}})\right|}{\left|(-\frac{1}{\sqrt{2}}+j\frac{1}{\sqrt{2}}-\frac{1}{2\sqrt{2}}-j\frac{1}{2\sqrt{2}})(-\frac{1}{\sqrt{2}}+j\frac{1}{\sqrt{2}}-\frac{1}{2\sqrt{2}}+j\frac{1}{2\sqrt{2}})\right|}$$

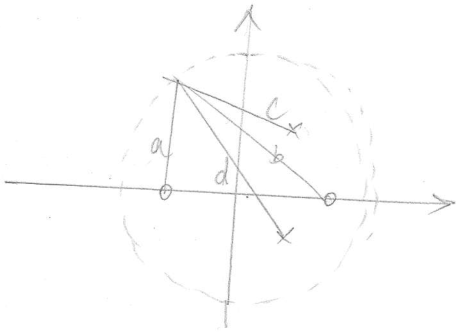
$$= \frac{2}{3}$$

$$\angle a = \arctan\left(\frac{\frac{1}{\sqrt{2}}}{-\frac{2}{\sqrt{2}}}\right) + \pi = 2.3564$$

$$\angle b = \frac{\pi}{2}$$

$$\angle c = \arctan\left(\frac{\frac{1}{\sqrt{2}}-\frac{1}{2\sqrt{2}}}{-\frac{1}{\sqrt{2}}-\frac{1}{2\sqrt{2}}}\right) + \pi = 2.8198$$

$$\angle d = \frac{3}{4}\pi$$



$$\angle H(e^{j\frac{3\pi}{4}}) = \angle a + \angle b - \angle c - \angle d = -0.9273$$

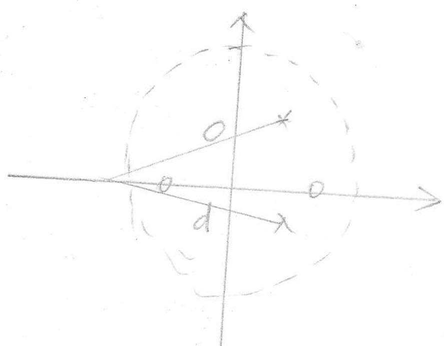
$\omega = \pi. \qquad z = e^{j\pi} = -1$

$$|H(e^{j\pi})| = |H(z=-1)| = \left|\frac{(-1+j\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}})(-1+j\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{2}})}{(-1-\frac{1}{2\sqrt{2}}-j\frac{1}{2\sqrt{2}})(-1-\frac{1}{2\sqrt{2}}+j\frac{1}{2\sqrt{2}})}\right|$$

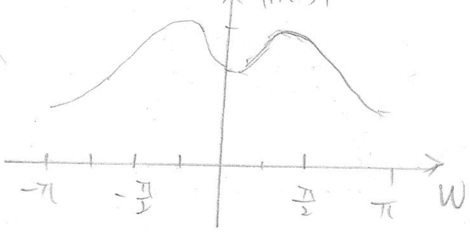
$$= 0.255$$

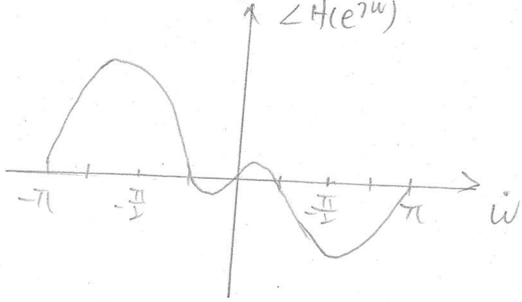
$$\angle H(e^{j\pi}) = \angle a + \angle b - \angle c - \angle d$$

$$= \pi + \pi = 2\pi.$$



$|H(\omega)|$ is even-symetric $\qquad \angle H(\omega)$ is odd-symetric





c. Stable. because system is casual. the ROC will extand outward from the outer most pole Since $|P_1| = |P_2| < 1$ all the poles are insider the unit circle. The ROC will contain the unit circle. Therefore, it is stable

d. $$\frac{Y(z)}{X(z)} = H(z) = \frac{1 - \frac{1}{2} z^{-2}}{1 - \frac{1}{\sqrt{2}} z^{-1} + \frac{1}{4} z^{-2}}$$

$$Y(z)\left(1 - \frac{1}{\sqrt{2}} z^{-1} + \frac{1}{4} z^{-2}\right) = X(z)\left(1 - \frac{1}{2} z^{-2}\right)$$

$$\leftrightarrow \quad y[n] - \frac{1}{\sqrt{2}} y[n-1] + \frac{1}{4} y[n-2] = x[n] - \frac{1}{2} x[n-2]$$

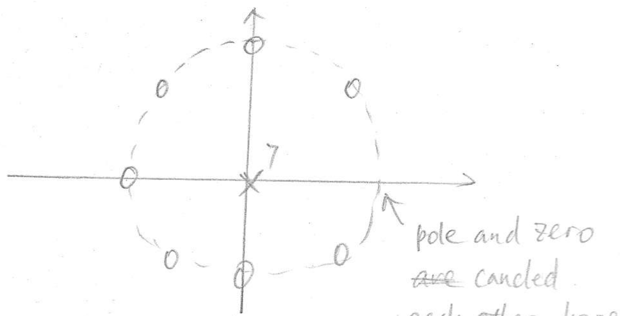
$$y[n] = x[n] - \frac{1}{2} x[n-2] + \frac{1}{\sqrt{2}} y[n-1] - \frac{1}{4} y[n-2]$$

3. a. $h[n] = \frac{1}{8} [\delta[n] + \cdots + \delta[n-7]]$

it is finite duration.

b. $$H(z) = \frac{1}{8} [1 + z^{-1} + \cdots + z^{-7}]$$

$$= \frac{1}{8} \frac{1 - z^{-8}}{1 - z^{-1}}$$



c. $$H(z) = \frac{1}{8} \frac{z^8 - 1}{z^7 (z-1)}$$

poles: $p_1 = 1 \quad p_2 = \cdots = p_8 = 0$

zeros: $z^8 - 1 = 0 \Rightarrow z_k = e^{j\frac{2\pi k}{8}}, \quad k = 0, \cdots, 7$

4. a. [illegible]

$$Y(z) = \frac{1}{8}\{X(z) - z^{-8}X(z)\} + z^{-1}Y(z)$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1}{8}\,\frac{1-z^{-8}}{1-z^{-1}} = \frac{1}{8}\,\frac{z^{8}-1}{z^{7}(z-1)}$$

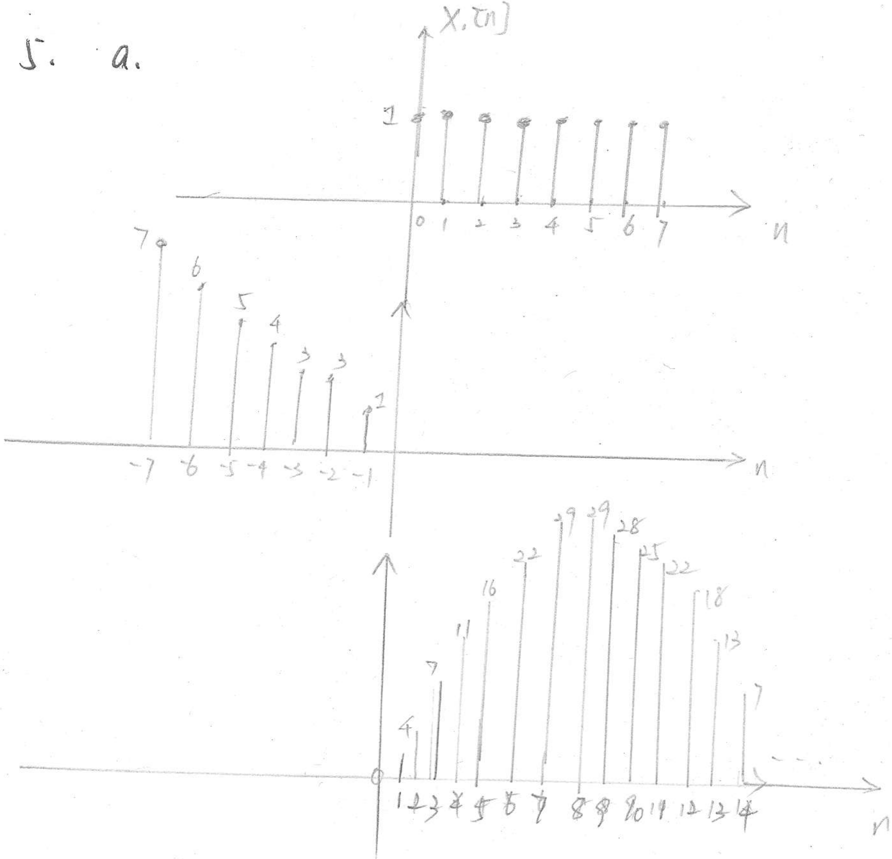
b. the same with part c. problem 3.

c. $H(z) = \frac{1}{8}(1+z^{-1}+ \cdots + z^{-8})$

$h[n] = \frac{1}{8}[\delta[n] + \delta[n-1] + \cdots + \delta[n-8]]$

It is finite duration

5. a.



$y[n] = \{\underset{\uparrow \, n=0}{0}, 1, 4, 7, 11, 16, 22, 29, 29, 28, 25, 22, 18, 13, 7\}$

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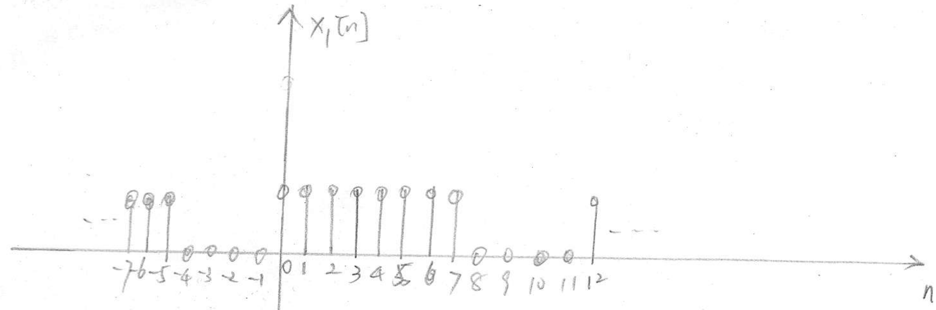
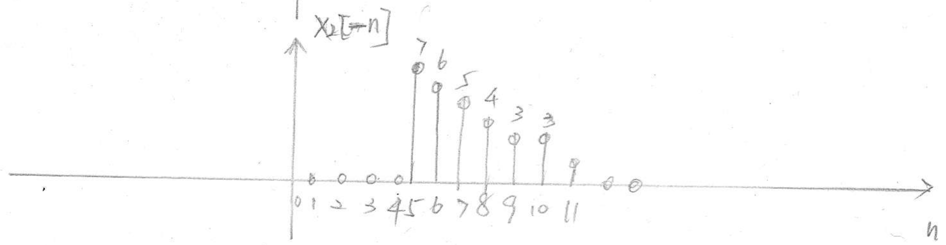
b.





In one period.

n	0	1	2	3	4	5	6	7	8	9	10	11
$x_1[n]$	1	1	1	1	1	1	1	1	0	0	0	0
$x_2[-n]$	0	0	0	0	0	7	6	5	4	3	3	1
$x_2[1-n]$	1	0	0	0	0	0	7	6	5	4	3	3
$x_2[2-n]$	3	1	0	0	0	0	0	7	6	5	4	3
$x_2[3-n]$	3	3	1	0	0	0	0	0	7	6	5	4
$x_2[4-n]$	4	3	3	1	0	0	0	0	0	7	6	5
$x_2[5-n]$	5	4	3	3	1	0	0	0	0	0	7	6
$x_2[6-n]$	6	5	4	3	3	1	0	0	0	0	0	7
$x_2[7-n]$	7	6	5	4	3	3	1	0	0	0	0	0
$x_2[8-n]$	0	7	6	5	4	3	3	1	0	0	0	0
$x_2[9-n]$	0	0	7	6	5	4	3	3	1	0	0	0
$x_2[10-n]$	0	0	0	7	6	5	4	3	3	1	0	0
$x_2[11-n]$	0	0	0	0	7	6	5	4	3	3	1	0

$y_{12}[n] =$

$\Rightarrow y_{12}[0] = 18$

$\Rightarrow y_{12}[1] = 14$

$y_{12}[2] = 11$

$y_{12}[3] = 7$

$y_{12}[4] = 11$

$y_{12}[5] = 16$

$y_{12}[6] = 22$

$y_{12}[7] = 29$

$y_{12}[8] = 29$

$y_{12}[9] = 28$

$y_{12}[10] = 25$

$y_{12}[11] = 22$

$$y_{12}[n] = x_1[n] \circledast_{12} x_2[n] = \{ \underset{\uparrow}{18}, 14, 11, 7, 11, 16, 22, 29, 29, 28, 25, 22 \}$$

c. Since both the sequences are of length 8, thus, length of sequence should be $8+8-1 = 15$

$\therefore$ padding of 7 zeros in each sequence is required.