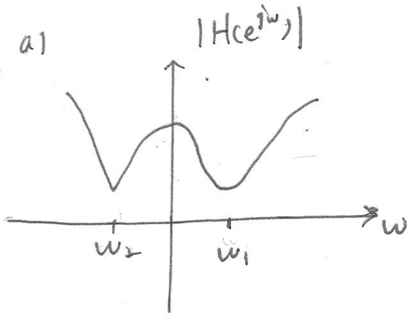
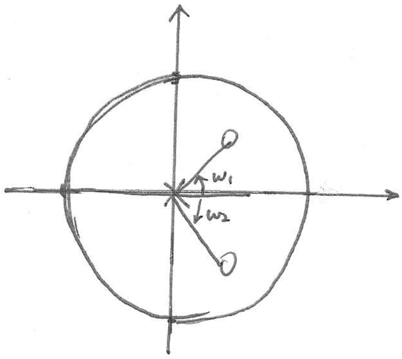
ECE 438 HW 2 Solution

1.

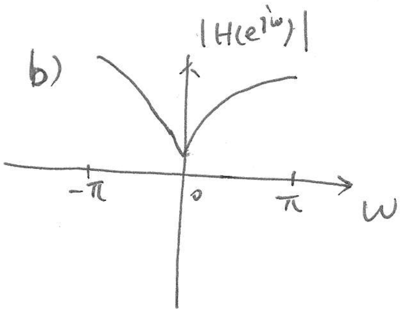
a)





a pole near the unit circle will cause the frequency response to increase in the neighborhood of the pole; a zero will cause the frequency response to decrease in the neighborhood of that zero.

b)



As to (b), we use a different method.

$$H(z) = \frac{z-a}{z}$$

$$H(e^{jw})\Big|_{z=e^{jw}} = \frac{e^{jw}-a}{e^{jw}} = 1-ae^{-jw}$$

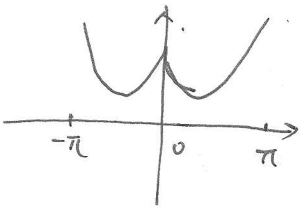
$$|H(e^{jw})| = \sqrt{(1-ae^{-jw})(1-a\ ^{jw})}$$

$$= \sqrt{1+r^2-2r\cos w}$$

it gets minimum, when $w=0$

it gets maximum, when $w=\pi$

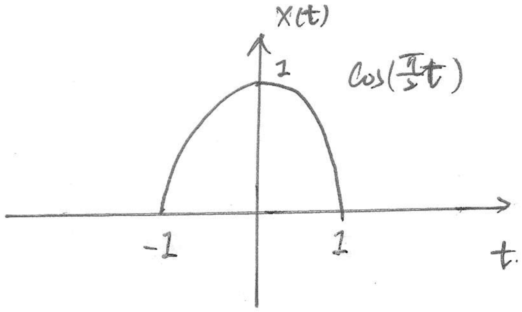
c)



There are 2 poles at $w=0, \pi$

Therefore, there should be two peaks at $w=$ π and 0.

1. (a) i.



$$x(t) = \cos(\tfrac{\pi}{2}t)\, \mathrm{rect}(\tfrac{t}{2})$$

$$\cos \tfrac{\pi}{2}t \longleftrightarrow \tfrac{1}{2}\left(\delta(f+\tfrac{1}{4}) + \delta(f-\tfrac{1}{4})\right)$$

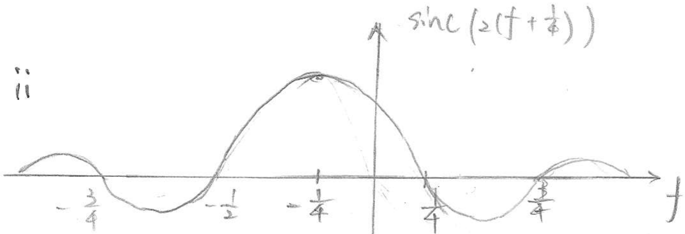
$$\mathrm{rect}\, \tfrac{t}{2} \longleftrightarrow 2\, \mathrm{sinc}(2f)$$

$$\Rightarrow X(f) = \tfrac{1}{2}\left(\delta(f+\tfrac{1}{4}) + \delta(f-\tfrac{1}{4})\right) * 2\,\mathrm{sinc}(2f)$$

$$= \mathrm{sinc}\left(2(f+\tfrac{1}{4})\right) + \mathrm{sinc}\left(2(f-\tfrac{1}{4})\right)$$

ii

$$\mathrm{sinc} f = \frac{\sin \pi f}{\pi f}$$



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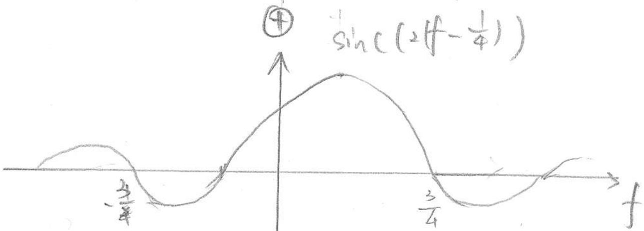

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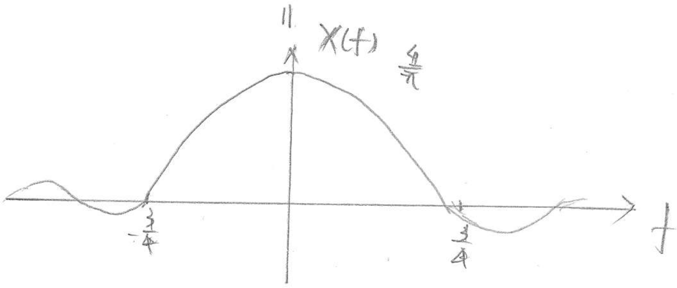
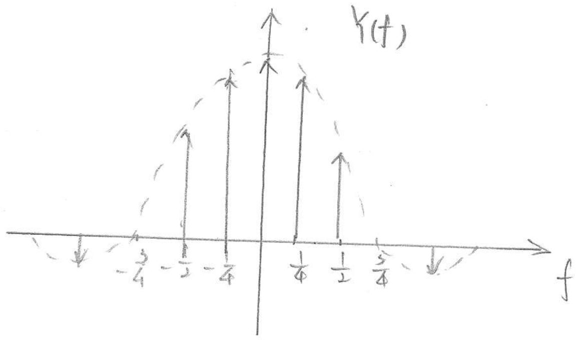


(b) $y(t) = \text{rept}_4\,(x(t))$

$\Rightarrow\ y(t) = \text{rept}_4\,(x(t)) \longleftrightarrow Y(f) = \frac{1}{4}\,\text{comb}_{\frac{1}{4}}\,(X(f))$

$$= \frac{1}{4}\sum_{k=-\infty}^{\infty} X\left(\frac{1}{4}f\right)\delta\left(f-\frac{1}{4}k\right)$$

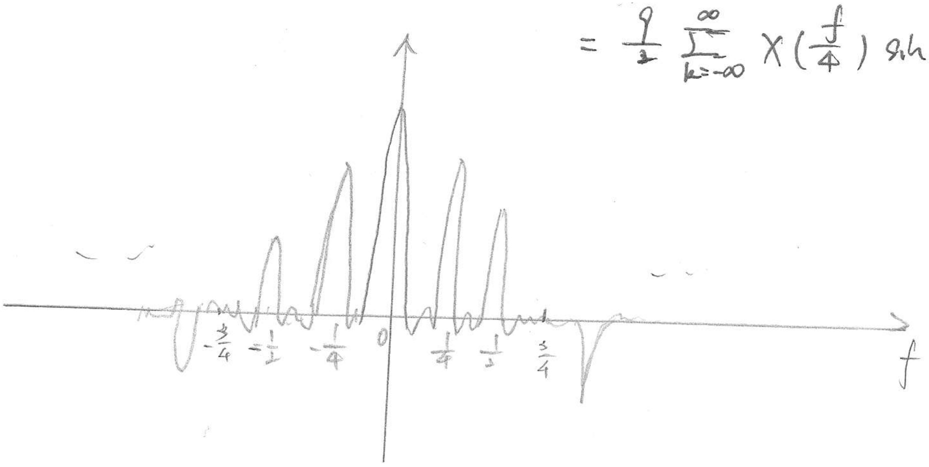


(c) $z(t) = y(t)\,\text{rect}\left(\frac{t}{18}\right)$

$\Rightarrow\ z(t) = y(t)\,\text{rect}\left(\frac{t}{18}\right) \longleftrightarrow Z(f) = Y(f) * 18\,\text{sinc}(18f)$

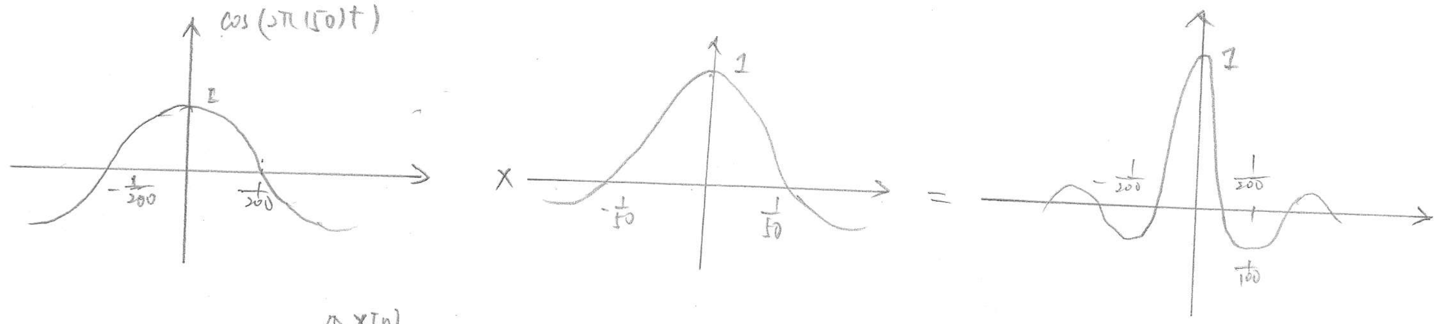
$$= \frac{1}{4}\sum_{k=-\infty}^{\infty} x\left(\frac{f}{4}\right)\left\{\delta\left(f-\frac{k}{4}\right) * 18\,\text{sinc}(18f)\right\}$$

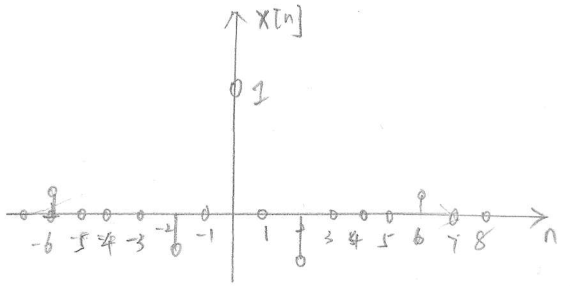
$$= \frac{9}{2}\sum_{k=-\infty}^{\infty} X\left(\frac{f}{4}\right)\text{sinc}\left(18\left(f-\frac{k}{4}\right)\right)$$



3. a. $x[n] = x(t)\Big|_{t=nT} = \cos(2\pi(50)\,0.005n)\,\mathrm{sinc}(50\times 0.005n)$

$= \cos\frac{\pi}{2}n\ \mathrm{sinc}(0.25n)$





b. $x(t) = \frac{1}{2}\left[e^{-j2\pi 50t} + e^{j2\pi 50t}\right]\cdot \mathrm{sinc}(50t)$

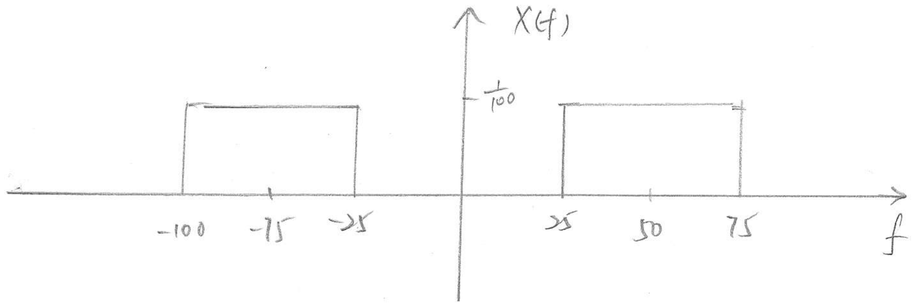
$e^{+j2\pi 50t} \longleftrightarrow \delta(f-50)$

$e^{-j2\pi 50t} \longleftrightarrow \delta(f+50)$

$\mathrm{sinc}(50t) \longleftrightarrow \frac{1}{50}\,\mathrm{rect}\left(\frac{1}{50}f\right)$

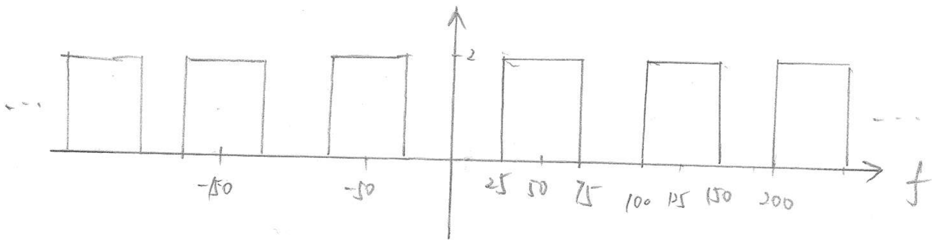
$\Rightarrow X(f) = \frac{1}{2}\left[\delta(f-50) + \delta(f+50)\right] * \frac{1}{50}\,\mathrm{rect}\left(\frac{1}{50}f\right)$

$= \frac{1}{100}\left(\mathrm{rect}\left(\frac{f-50}{50}\right) + \mathrm{rect}\left(\frac{f+50}{50}\right)\right)$

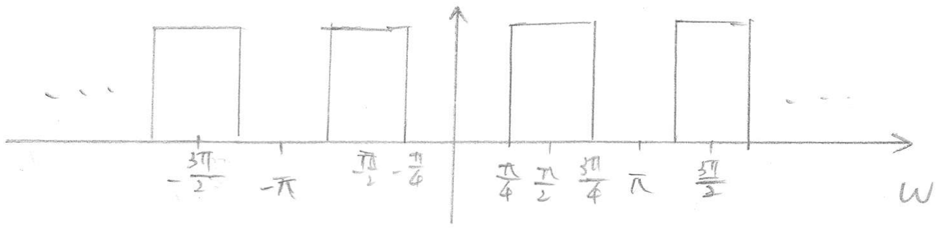


c. $X_s(t) = \text{comb}_T(x(t))$

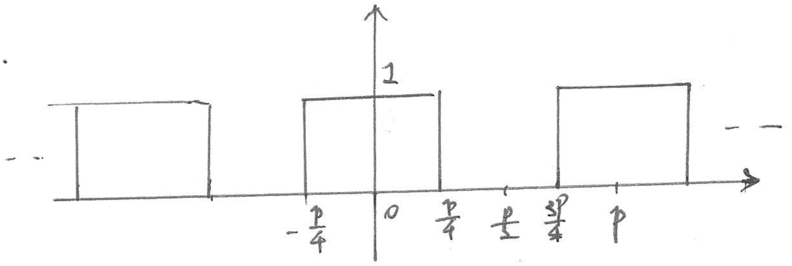
$$X_s(f) = \frac{1}{T}\,\text{rep}_{\frac{1}{T}}(X(f)) = 200 \sum_{k=-\infty}^{\infty} X(f-200k)$$



d. $X(\omega) = X_s(f)\Big|_{f=\frac{\omega}{2\pi T}}$



4.

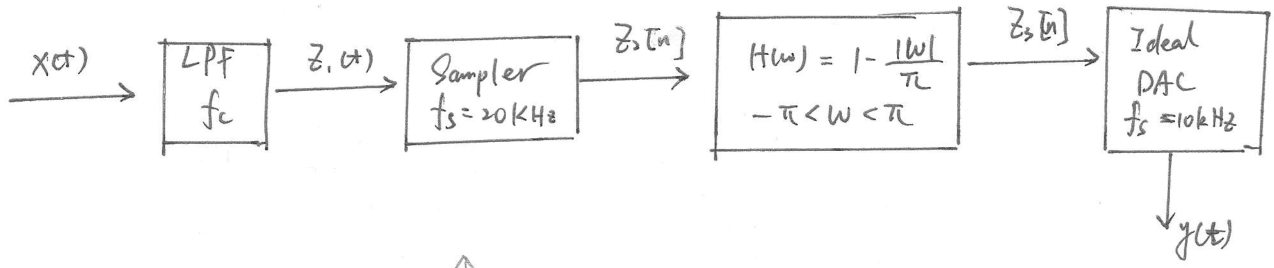


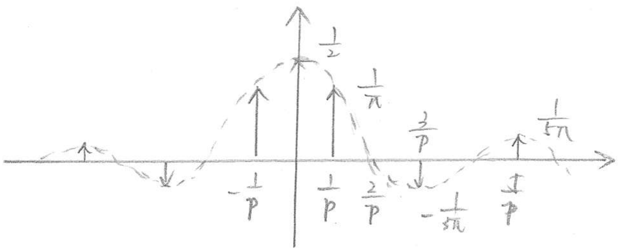
$$x(t) = \text{rep}_P \left\{ \text{rect}\left(\frac{t}{P/2}\right) \right\}$$

$$\Updownarrow$$

$$X(f) = \frac{1}{P} \text{comb}_{\frac{1}{P}} \left\{ \frac{P}{2} \text{sinc}\left(\frac{P}{2} f\right) \right\}$$

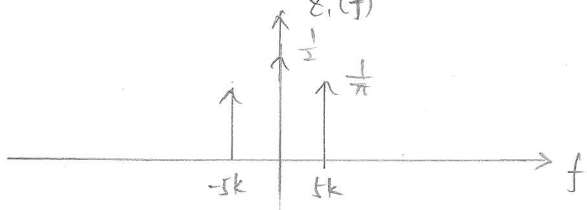
$$= \frac{1}{2} \text{comb}_{\frac{1}{P}} \left\{ \text{sinc}\left(\frac{P}{2} f\right) \right\}$$



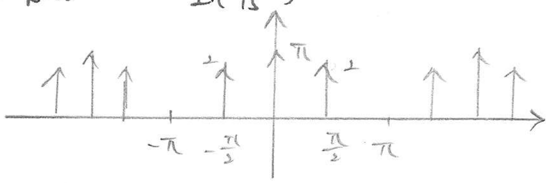


(a) $P = 0.2$ ms, $f_c = 10$ kHz.

$Z_1(f)$ is



$$Z_2(\omega) = \frac{1}{T_s} \sum_{k=-\infty}^{\infty} Z_1\left(\frac{\omega - 2\pi k}{2\pi T_s}\right)$$



to calculate amplitude

$$\delta\left(\frac{w-2\pi k}{2\pi T_s}\right) = 2\pi T_s\, \delta(w-2\pi k)$$

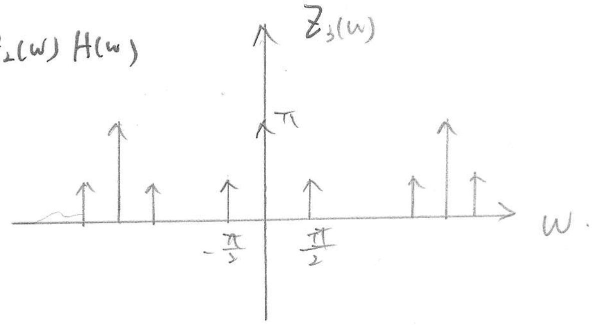
$$Z_2(w=0) = \frac{1}{T_s}\cdot\frac{1}{2}(2\pi T_s)\,\delta(0)$$

$$= \pi\,\delta(0)$$

$$Z_2\left(w=\frac{\pi}{2}\right) = \frac{1}{T_s}\frac{1}{\pi}(2\pi T_s)\,\delta\left(\frac{\pi}{2}\right)$$

$$= 2\delta\left(\frac{\pi}{2}\right)$$

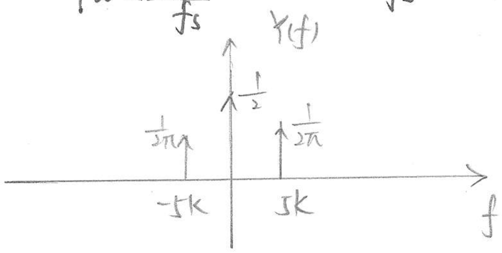
$Z_3(w) = Z_2(w)\,H(w)$



$Z_3(w=0) = \pi\delta($

$Z_3(w=\frac{\pi}{2}) = 2\delta($ $\quad \frac{1}{2} = \delta($

$Y(f) = Z_3(w)\Big|_{w=\frac{2\pi f}{f_s}}$ $\quad f_s = 20kHz$

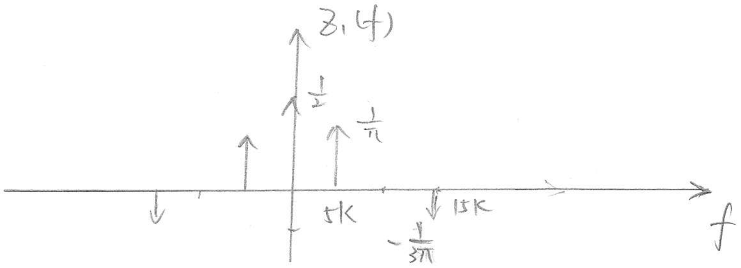


$$Y(f) = \frac{1}{2}\delta(f) + \frac{1}{2\pi}\left(\delta(f-5000)+\delta(f+5000)\right)$$

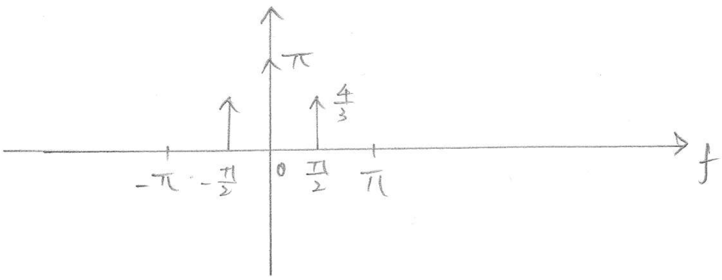
$$y(t) = \frac{1}{2} + \frac{1}{\pi}\cos(2\pi(5000t))$$

(b) $P = 0.2\,ms$ $\quad f_c = 20\,kHz$.

$Z_1(f)$



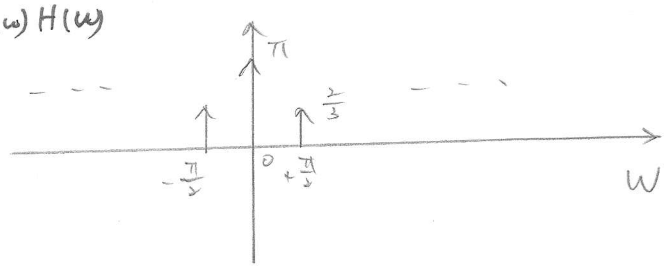
$$Z_2(\omega) = \frac{1}{T_s} \sum_{k=-\infty}^{\infty} Z_1\left(\frac{\omega - 2\pi k}{2\pi T_s}\right)$$



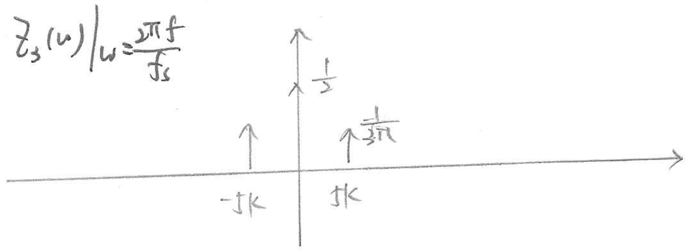
$$Z_2(\omega=0) = \frac{1}{T_s}\frac{1}{2}(2\pi T_s)\delta(\omega=0)$$
$$= \pi\delta(\omega=0)$$

$$Z_2(\omega=\tfrac{\pi}{2}) = \frac{1}{T_s}\frac{1}{\pi}(2\pi T_s)\delta(\omega-\tfrac{\pi}{2})$$
$$+ \frac{1}{T_s}(-\tfrac{1}{3\pi})(2\pi T_s)\delta(\omega-\tfrac{\pi}{2})$$
$$= (2-\tfrac{2}{3})\delta(\omega-\tfrac{\pi}{2})$$
$$= \tfrac{4}{3}\delta(\omega-\tfrac{\pi}{2})$$

$Z_3(\omega) = Z_2(\omega)H(\omega)$



$Y(f) = Z_3(\omega)\big|_{\omega=\frac{2\pi f}{f_s}}$

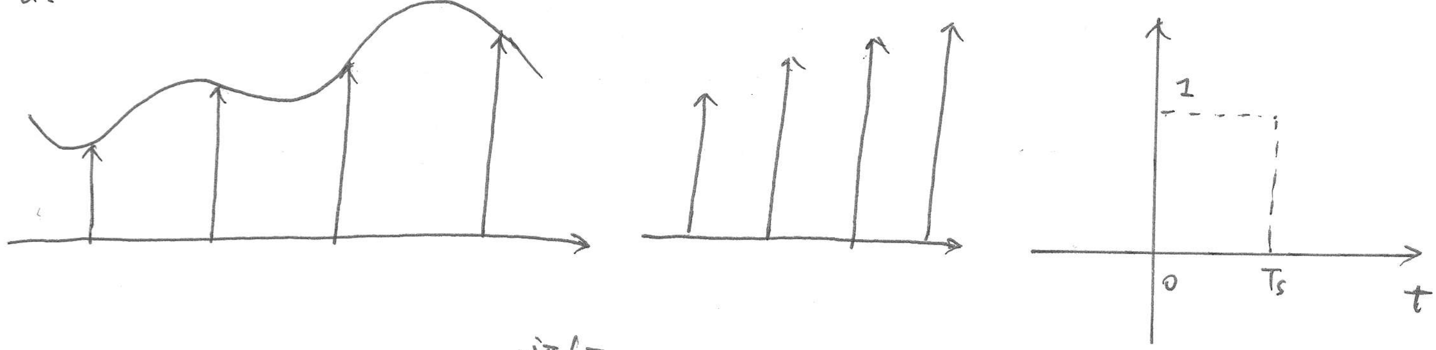


$$Y(f) = \tfrac{1}{2}\delta(f) + \frac{1}{3\pi}\left(\delta(f-5000) + \delta(f+5000)\right)$$

$$y(t) = \tfrac{1}{2} + \frac{2}{3\pi}\cos(2\pi(5000t))$$

5. a.

$x_{ZOH}(t) = X_s(t) * g(t)$



$$X_{ZOH}(f) = X_s(f)\, T_s e^{-j\pi f T_s}\, \mathrm{sinc}(f T_s)$$

$$X_s(f) = \frac{1}{T_s} \sum_{n=-\infty}^{\infty} X_a\left(f - \frac{n}{T_s}\right)$$

$$\Rightarrow X_{ZOH}(f) = \cancel{\frac{1}{T_s}} \sum_{n=-\infty}^{\infty} X_a\left(f - \frac{n}{T_s}\right) \cancel{T_s} e^{-j\pi f T_s}\, \mathrm{sinc}(f T_s)$$

$$= e^{-j\pi f T_s}\, \mathrm{sinc}(f T_s) \sum_{n=-\infty}^{\infty} X_a\left(f - \frac{n}{T_s}\right)$$

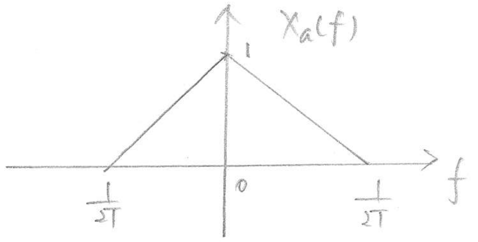
sinc(ωf)

sin(T_sf)   2πf

$T = \frac{2\pi}{2\pi} = 1$ ω

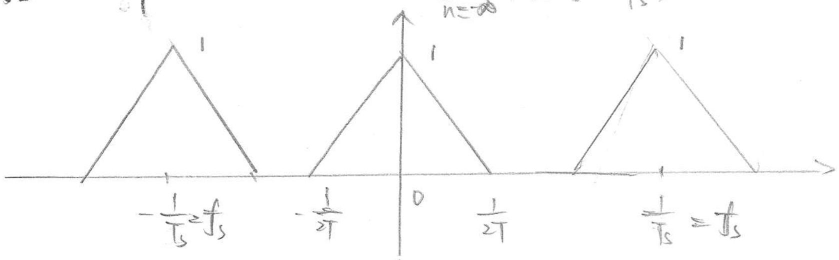
$T = \frac{2\pi}{T_s}$

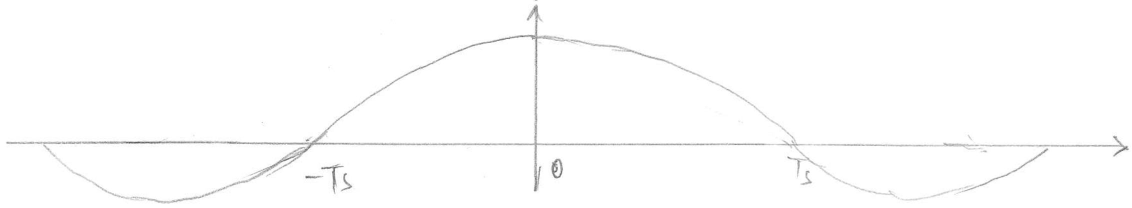
b.

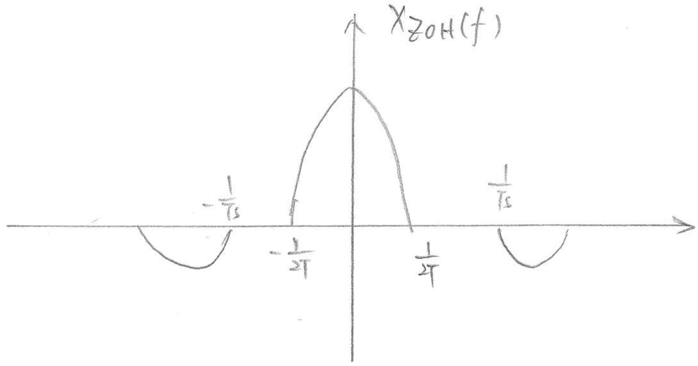


If $f_s > \frac{1}{T}$

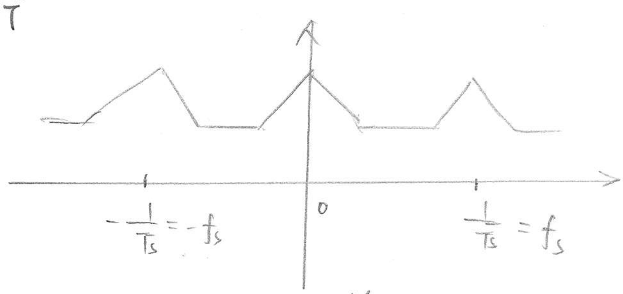
$\sum_{n=-\infty}^{\infty} X_a\left(f - \frac{n}{T_s}\right)$

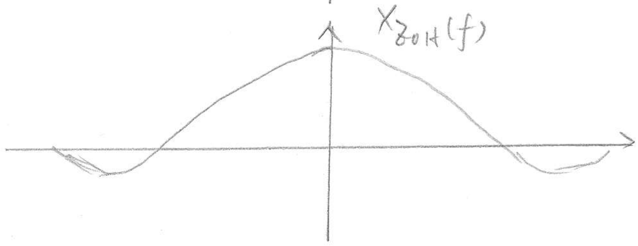






if $f_s < \frac{1}{T}$



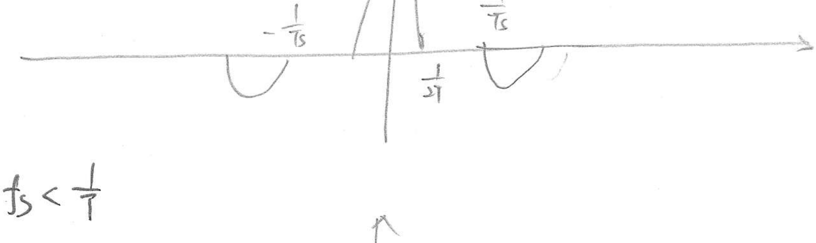


c.

$$X_{FOH}(f) = X_s(f)\, T_s\, e^{-j\pi f T_s} \operatorname{sinc}^2(f T_s)$$

$$= e^{-j\pi f T_s} \operatorname{sinc}^2(f T_s) \sum_{n=-\infty}^{\infty} X_a\left(f - \frac{n}{T_s}\right)$$

if $f_s > \frac{1}{T}$



if $f_s < \frac{1}{T}$